Math 115

Spring 2018

Lecture 22

$? \; a^2 + b^2 = c^2 \; ?$

$y = mx + b \quad ? \quad d = rt$

Class Quiz

① Simplify: $(-4x^5)^3 = (-4)^3(x^5)^3 = \boxed{-64x^{15}}$

② Simplify: $\left(\frac{-5x^{-2}}{y^{-4}}\right)^{-2} = \left(\frac{-5y^4}{x^2}\right)^{-2} = \left(\frac{x^2}{-5y^4}\right)^2 = \boxed{\frac{x^4}{25y^8}}$

③ Divide: $\frac{x^3 + 5x^2 - 6}{x - 1}$

$x \boxed{x^2} = x^3$

$x \boxed{6x} = 6x^2$

$x \boxed{6} = 6x$

$\boxed{x^2 + 6x + 6}$

$$
\begin{array}{r|l}
 & x^2 + 6x + 6 \\
\hline
x - 1 & x^3 + 5x^2 + 0x - 6 \\
 & -(x^3 - x^2) \\
\hline
 & 6x^2 + 0x - 6 \\
 & -(6x^2 - 6x) \\
\hline
 & 6x - 6 \\
 & -(6x - 6) \\
\hline
 & 0
\end{array}
$$

Find the missing factor:

① $18x^3 = 3x^2 \cdot (6x)$

② $18x^3 - 15x = 3x(6x^2 - 5)$

③ $-4x^2(8x-1) + 7x(8x-1) - 100(8x-1) =$

$(8x-1)(-4x^2 + 7x - 100)$

Factor completely:

① $14x^2 + 49x = 7x(2x + 7)$

(7 · 2, 7 · 7)

② $25x^2(3x-4) - 8x(3x-4) + 50(3x-4)$

$= (3x-4)(25x^2 - 8x + 50)$

③ $4x^3 - 5x^2 + 40x - 50$

$= x^2(4x - 5) + 10(4x - 5)$

$= (4x - 5)(x^2 + 10)$

④ $x^2 + 7x + 12 = \boxed{(x+3)(x+4)}$

1·12
2·6
3·4

⑤ $x^2 - 7x + 12 = (x-3)(x-4)$

⑥ $x^2 - x - 12 = (x+3)(x-4)$

⑦ $x^2 + x - 12 = (x+4)(x-3)$

⑧ $x^3 + 5x^2 - 14x = x(x^2 + 5x - 14)$

$= \boxed{x(x+7)(x-2)}$

GCF

1·14
2·7

⑨ $2x^3 - 12x^2 - 32x$

1·16
2·8
4·4

$= 2x(x^2 - 6x - 16)$

$= \boxed{2x(x+2)(x-8)}$

Factor completely:

$2x^2 - x - 6 = 2x^2 + 3x - 4x - 6$

$P = -12$

$S = -1$

-12

$1 \cdot 12$

$2 \cdot 6$

$3 \cdot 4$

$= x(2x+3) - 2(2x+3)$

$= (2x+3)(x-2)$

$2xy - 8y + 3x - 12$

$= 2y(x-4) + 3(x-4) = (x-4)(2y+3)$

$3x^2 + 11x - 4 = 3x^2 + 12x - x - 4$

$P = -12$

$S = 11$

-12

$-1 \cdot 12$

$-2 \cdot 6$

$-3 \cdot 4$

$= 3x(x+4) - 1(x+4)$

$= (x+4)(3x-1)$

$6x^2 - 5x - 6$ $= 6x^2 + 4x - 9x - 6$

$P = -36$, $S = -5$, -36

$1 \cdot -36$
$2 \cdot -18$
$3 \cdot -12$
$4 \cdot -9$
$6 \cdot -6$

$= 2x(3x+2) - 3(3x+2)$

$= (3x+2)(2x-3)$

$24x^5 - 20x^4 + 4x^3$

$= 4x^3(6x^2 - 5x + 1) =$ $6x^2 - 2x - 3x + 1$

$P = 6$, $S = -5$, 6

$-1 \cdot -6$
$-2 \cdot -3$

$2x(3x-1) - 1(3x-1)$

$4x^3(3x-1)(2x-1)$

$3x^2 + 5x - 24xy - 40y$

$= x(3x+5) - 8y(3x+5) = (3x+5)(x-8y)$

$4x^2 - 20x + 25$ $= 4x^2 - 10x - 10x + 25$

$P = 100$, $S = -20$, 100, $-10, -10$

$= 2x(2x-5) - 5(2x-5)$

$= (2x-5)(2x-5)$

$= (2x-5)^2$

$2x^2 - 15 - 7x$

$= 2x^2 - 7x - 15$ $= 2x^2 + 3x - 10x - 15$

$P = -30$, $S = -7$, -30, $3, -10$

$= x(2x+3) - 5(2x+3)$

$= (2x+3)(x-5)$

Special Factoring:

Binomials (Two Terms)

$A^2 + B^2 \rightarrow$ Sum of two Squares

$A^2 - B^2 \rightarrow$ Difference of two Squares

$A^3 + B^3 \rightarrow$ Sum of two Cubes

$A^3 - B^3 \rightarrow$ Difference of two Cubes

$A^2 + B^2 = \text{Prime}$

$A^2 - B^2 = (A+B)(A-B)$

$x^2 + 100 = x^2 + 10^2 = \text{Prime}$

$x^2 - 100 = x^2 - 10^2 = (x+10)(x-10)$

$4x^2 + 49 = (2x)^2 + (7)^2 = \text{Prime}$

$4x^2 - 49 = (2x)^2 - (7)^2 = (2x+7)(2x-7)$

$25x^2 + 64 = (5x)^2 + (8)^2$ Prime

$25x^2 - 64 = (5x)^2 - (8)^2 = (5x+8)(5x-8)$

$36x^2 + 121y^2 = (6x)^2 + (11y)^2$ Prime

$36x^2 - 121y^2 = (6x)^2 - (11y)^2$

$= (6x - 11y)(6x + 11y)$

Factor Completely:

① $x^3 - 25x = x(x^2 - 25)$

$= x(x+5)(x-5)$

② $2x^4 - 72x^2 = 2x^2(x^2 - 36)$

$= 2x^2(x+6)(x-6)$

③ $25x^3y - 81xy^3$

$= xy(25x^2 - 81y^2) = xy(5x+9y)(5x-9y)$

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

$$x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$$

$$x^3 - 27 = x^3 - 3^3 = (x - 3)(x^2 + 3x + 9)$$

$$x^3 + 125 = x^3 + 5^3$$

$$= (x + 5)(x^2 - 5x + 25)$$

$$x^3 - 1000 = x^3 - 10^3$$

$$= (x - 10)(x^2 + 10x + 100)$$

$27x^3 + 64y^3$

$= (3x)^3 + (4y)^3$

$= (3x + 4y)(9x^2 - 12xy + 16y^2)$

$125x^3 - 8y^3$

$= (5x)^3 - (2y)^3 = (5x - 2y)(25x^2 + 10xy + 4y^2)$

Special Factoring

Trinomials

$A^2 + 2AB + B^2 = (A + B)^2$

$A^2 - 2AB + B^2 = (A - B)^2$

Perfect - Square Trinomials

$25x^2 + 70x + 49$

$= (5x + 7)^2$

$2 \cdot 5x \cdot 7$

$64x^2 - 80x + 25$

$= (8x - 5)^2$

$36x^2 + 60xy + 25y^2$

$= (6x + 5y)^2$

$49x^3 - 140x^2y + 100xy^2$

$= x[49x^2 - 140xy + 100y^2]$

$= x(7x - 10y)^2$

Factor Completely:

① $7x + 14$

② $6y^2 + 9y + 4xy + 6x$

③ $x^2 - x - 56$

④ $2x^2 + 5x + 2$